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-> kill(all)$
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COMBINATION : CIRCLE AND LINE

peter.vlasschaert@gmail.com,10/07/2017 version 1 : math "circle and line" equations : lines p1,p2 => m,n (implicit) circle p3,p4 => a,b,r (explicit)

-> p1:y=m1*x+n1; p2:y=m2*x+n2; p3:(x-a1)^2+(y-b1)^2=r1^2; p4:(x-a2)^2+(y-b2)^2=r2^2;

$$y = m1x + n1 \tag{\%o1}$$

$$y = m2x + n2 \tag{\%o2}$$

$$(y - b1)^2 + (x - a1)^2 = r1^2 \tag{\%o3}$$

$$(y - b2)^2 + (x - a2)^2 = r2^2 \tag{\%o4}$$

line p1,line p2

-> q1:part(p1,2)=part(p2,2); q2:part(solve([q1], [x]),1);

$$m1x + n1 = m2x + n2 \tag{\%o5}$$

$$x = -\frac{n2 - n1}{m2 - m1} \tag{\%o6}$$

subst : q2 in p1 rem : abs(m2-m1) <> 0 for intersection

-> q3:ratsimp(ev(p1,q2));

$$y = -\frac{m1\ n2 - m2\ n1}{m2 - m1} \tag{\%o7}$$

line p1,circle p3 a) find : intersection (p1,p3)

-> q4:ev(p3,p1); q5:lhs(q4)-rhs(q4)=0; q6:expand(q5);

$$(m1x + n1 - b1)^2 + (x - a1)^2 = r1^2 \tag{\%o8}$$

$$(m1x + n1 - b1)^2 + (x - a1)^2 - r1^2 = 0 \tag{\%o9}$$

$$m1^2\,x^2 + x^2 + 2m1\ n1x - 2b1\ m1x - 2a1x - r1^2 + n1^2 - 2b1\ n1 + b1^2 + a1^2 = 0 \tag{\%o10}$$

b)find: a*x^2+b*x+c=0 a=>q7a b=>q7b c=>q7c

-> q7a: part(coeff(q6,x,2),1); q7b:part(coeff(q6,x,1),1); q7c:part(coeff(q6,x,0),1);

$$m1^2 + 1 \tag{\%o11}$$

$$2m1\ n1 - 2b1\ m1 - 2a1 \tag{\%o12}$$

$$-r1^2 + n1^2 - 2b1\ n1 + b1^2 + a1^2 \tag{\%o13}$$

-> q8: q7a*x^2+q7b*x+q7c=0; q9:solve([q8: q7a*x^2+q7b*x+q7c=0], [x]);

$$\left(m1^2 + 1\right) x^2 + (2m1\ n1 - 2b1\ m1 - 2a1)\,x - r1^2 + n1^2 - 2b1\ n1 + b1^2 + a1^2 = 0 \tag{1}$$

$$x = -\frac{\sqrt{\left(m1^2+1\right)\,r1^2 - n1^2 + \left(2b1 - 2a1\,m1\right)\,n1 - a1^2\,m1^2 + 2a1\,b1\,m1 - b1^2} + m1\,n1 - b1\,m1 - a1}{m1^2+1}, x$$
$$= \frac{\sqrt{\left(m1^2+1\right)\,r1^2 - n1^2 + \left(2b1 - 2a1\,m1\right)\,n1 - a1^2\,m1^2 + 2a1\,b1\,m1 - b1^2} - m1\,n1 + b1\,m1 + a1}{m1^2+1} \tag{2}$$

c)find : (m1^2+1)*r1^2-n1^2+(2*b1-2*a1*m1)*n1-a1^2*m1^2+2*a1*b1*m1-b1^2 >= 0 => real solution circle p3,circle p4 solve: system(p3,p4) a) -1*p3

-> q10:-1*expand(p3); q10a:expand(p3); q11:expand(p4);

$$-y^2 + 2b1\,y - x^2 + 2a1\,x - b1^2 - a1^2 = -r1^2 \tag{\%o16}$$

$$y^2 - 2b1\,y + x^2 - 2a1\,x + b1^2 + a1^2 = r1^2 \tag{\%o17}$$

$$y^2 - 2b2\,y + x^2 - 2a2\,x + b2^2 + a2^2 = r2^2 \tag{\%o18}$$

b) add(q10,q11) result : a*x+b*y +c = 0 a = q13a b = q13b c = q13c rem : y= -a/b*x -c/b , m = -a/b , n = -c/b

-> q12:q11+q10$ q13:rhs(q12)-lhs(q12)=0;

$$2b2\,y - 2b1\,y + 2a2\,x - 2a1\,x + r2^2 - r1^2 - b2^2 + b1^2 - a2^2 + a1^2 = 0 \tag{\%o20}$$

-> q13a : part(coeff(q13,x,1),1); q13b :part(coeff(q13,y,1),1); q13c :ev(part(q13,1),x=0,y=0);

$$2a2 - 2a1 \tag{\%o21}$$

$$2b2 - 2b1 \tag{\%o22}$$

$$r2^2 - r1^2 - b2^2 + b1^2 - a2^2 + a1^2 \tag{\%o23}$$

-> q14:q13a*x+q13b*y+q13c=0;

$$\left(2b2 - 2b1\right)y + \left(2a2 - 2a1\right)x + r2^2 - r1^2 - b2^2 + b1^2 - a2^2 + a1^2 = 0 \tag{\%o24}$$

-> q14lm:m=-q13a/q13b; q14ln:n=-q13c/q13b;

$$m = \frac{2a1 - 2a2}{2b2 - 2b1} \tag{\%o25}$$

$$n = \frac{-r2^2 + r1^2 + b2^2 - b1^2 + a2^2 - a1^2}{2b2 - 2b1} \tag{\%o26}$$

c) solve q14 for x

-> q15:part(solve([q14], [x]),1);

$$x = -\frac{\left(2b2 - 2b1\right)y + r2^2 - r1^2 - b2^2 + b1^2 - a2^2 + a1^2}{2a2 - 2a1} \tag{\%o27}$$

d) insert x into q10a,numerator(q17)=num. ratio = num/denom

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-> q16:ratsimp(ev(q10a,q15))$ q17: part(q16,1,1)=0;
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$$\begin{aligned}&\left(4b2^2-8b1\ b2+4b1^2+4a2^2-8a1\ a2+4a1^2\right)y^2+\left((4b2-4b1)\ r2^2+(4b1-4b2)\ r1^2\right.\\&\left.-4b2^3+4b1\ b2^2+\left(4b1^2-4a2^2+8a1\ a2-4a1^2\right)\ b2-4b1^3+\left(-4a2^2+8a1\ a2-4a1^2\right)\ b1\right)y\\&+r2^4+\left(-2r1^2-2b2^2+2b1^2-2a2^2+4a1\ a2-2a1^2\right)\ r2^2+r1^4\\&+\left(2b2^2-2b1^2+2a2^2-4a1\ a2+2a1^2\right)\ r1^2+b2^4+\left(-2b1^2+2a2^2-4a1\ a2+2a1^2\right)\ b2^2\\&+b1^4+\left(2a2^2-4a1\ a2+2a1^2\right)\ b1^2+a2^4-4a1\ a2^3+6a1^2\ a2^2-4a1^3\ a2+a1^4=0\end{aligned}\tag{3}$$

e) find : coef's aa*y^2+bb*y+cc = 0 aa =q17a bb =q17b cc =q17c

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-> q17a:part(coeff(q17,y,2),1); q17b:part(coeff(q17,y,1),1); q17c:ev(part(q17,1),y=0);
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$$4b2^2-8b1\ b2+4b1^2+4a2^2-8a1\ a2+4a1^2\tag{\%o30}$$

$$\begin{aligned}&(4b2-4b1)\ r2^2+(4b1-4b2)\ r1^2-4b2^3+4b1\ b2^2\\&+\left(4b1^2-4a2^2+8a1\ a2-4a1^2\right)\ b2-4b1^3+\left(-4a2^2+8a1\ a2-4a1^2\right)\ b1\end{aligned}\tag{4}$$

$$\begin{aligned}&r2^4+\left(-2r1^2-2b2^2+2b1^2-2a2^2+4a1\ a2-2a1^2\right)\ r2^2+r1^4\\&+\left(2b2^2-2b1^2+2a2^2-4a1\ a2+2a1^2\right)\ r1^2+b2^4+\left(-2b1^2+2a2^2-4a1\ a2+2a1^2\right)\ b2^2\\&+b1^4+\left(2a2^2-4a1\ a2+2a1^2\right)\ b1^2+a2^4-4a1\ a2^3+6a1^2\ a2^2-4a1^3\ a2+a1^4\end{aligned}\tag{5}$$

f)find : two solutions ,one sol,two sol,no sol

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-> q18:solve(q17,y)$ q18a:part(q18,1); q18b:part(q18,2);
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$$\begin{aligned}y=-&((b2-b1+\%ia2-\%ia1)\ r2^2+(-b2+b1-\%ia2+\%ia1)\ r1^2-b2^3+(b1-\%ia2+\%ia1)\ b2^2\\&+\left(b1^2+(2\%ia2-2\%ia1)\ b1-a2^2+2a1\ a2-a1^2\right)\ b2-b1^3+(\%ia1-\%ia2)\ b1^2\\&+\left(-a2^2+2a1\ a2-a1^2\right)\ b1-\%ia2^3+3\%ia1\ a2^2-3\%ia1^2\ a2\\&+\%ia1^3)/(2b2^2-4b1\ b2+2b1^2+2a2^2-4a1\ a2+2a1^2)\end{aligned}\tag{6}$$

$$\begin{aligned}y=-&((b2-b1-\%ia2+\%ia1)\ r2^2+(-b2+b1+\%ia2-\%ia1)\ r1^2-b2^3+(b1+\%ia2-\%ia1)\ b2^2\\&+\left(b1^2+(2\%ia1-2\%ia2)\ b1-a2^2+2a1\ a2-a1^2\right)\ b2-b1^3+(\%ia2-\%ia1)\ b1^2\\&+\left(-a2^2+2a1\ a2-a1^2\right)\ b1+\%ia2^3-3\%ia1\ a2^2+3\%ia1^2\ a2\\&-\%ia1^3)/(2b2^2-4b1\ b2+2b1^2+2a2^2-4a1\ a2+2a1^2)\end{aligned}\tag{7}$$

q18a : y ->x (from q15) q18b : y ->x(from q15)

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-> q19a:ev(q15,q18a); q19b:ev(q15,q18b);
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$$\begin{aligned}x=-&(-((2b2-2b1)\,((b2-b1+\%ia2-\%ia1)\\&r2^2+(-b2+b1-\%ia2+\%ia1)\ r1^2-b2^3+(b1-\%ia2+\%ia1)\ b2^2+\left(b1^2+(2\%ia2-2\%ia1)\ b1-a2^2+2a1\ a2-a1^2\right)\ b2-b1^3+(\%ia1-\\&+r2^2-r1^2-b2^2+b1^2-a2^2+a1^2)/(2a2-2a1)\end{aligned}\tag{8}$$

$$\begin{aligned}x=-&(-((2b2-2b1)\,((b2-b1-\%ia2+\%ia1)\\&r2^2+(-b2+b1+\%ia2-\%ia1)\ r1^2-b2^3+(b1+\%ia2-\%ia1)\ b2^2+\left(b1^2+(2\%ia1-2\%ia2)\ b1-a2^2+2a1\ a2-a1^2\right)\ b2-b1^3+(\%ia2-\\&+r2^2-r1^2-b2^2+b1^2-a2^2+a1^2)/(2a2-2a1)\end{aligned}\tag{9}$$

example : circle1 and circle 2

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-> v1:a1=2$ v2:b1=3$ v3:r1=3$ v4:a2=1$ v5:b2=-1$ v6:r2=4$
-> z1:(-1)*expand(ev(p3,v1,v2,v3)); z2:expand(ev(p4,v4,v5,v6));
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$$-y^2+6y-x^2+4x-13=-9 \tag{\%o44}$$

$$y^2+2y+x^2-2x+2=16 \tag{\%o45}$$

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-> z3:z1+z2;
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$$8y+2x-11=7 \tag{\%o46}$$

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-> z4:lhs(z3)-rhs(z3)=0;
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$$8y+2x-18=0 \tag{\%o47}$$

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-> z5:part(solve(z4,x),1);
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$$x=9-4y \tag{\%o48}$$

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-> z6:(-1)*expand(ev(z1,z5));
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$$17y^2-62y+58=9 \tag{\%o49}$$

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-> z7:solve(z6,y);
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$$[y=-\frac{2^{\frac{7}{2}}-31}{17},y=\frac{2^{\frac{7}{2}}+31}{17}] \tag{\%o50}$$

y1,y2

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-> z7a:float(part(z7,1,2)); z7b:float(part(z7,2,2));
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$$1.158017147118543 \tag{\%o51}$$

$$2.489041676410868 \tag{\%o52}$$

x1,x2

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-> z8a:subst(z7a, y, z5); z8b:subst(z7b, y, z5);
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$$x=4.367931411525827 \tag{\%o53}$$

$$x=-0.956166705643474 \tag{\%o54}$$

two points (x1,y1) and (x2,y2)