07/02/2020

$$\int_{-\infty}^{+\infty} f(x)\, \delta(g(x))\, dx \qquad (1)$$

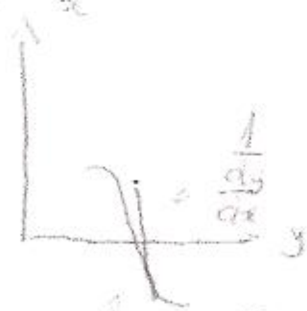
null points $g(x)$ $\quad (-\infty, +\infty)$

$g(x) = y \quad \rightarrow g(x_0) = 0 \qquad y = 0$

set : Nd $\quad y = g(x)$

$$\begin{cases} y = g(x) \\ g^{-1}(y) = g^{-1} g(x) \\ g^{-1}(y) = x \end{cases}$$

$dy = g'(x)\, dx$



same sign : slop = $\pm$ $\qquad$ slope = $\pm$

$$\pm \int_{g^{-1}(-\infty)}^{g^{-1}(x)} f(g^{-1}(y))\, \delta(y)\, \frac{dy}{g'(x)}$$

$\boxed{y = 0}$ $\qquad \pm \dfrac{f(g^{-1}(0))}{g'(x_0)} = \dfrac{f(x_0)}{|g'(x_0)|}$

REM: $\int f(x)\, \delta'(x)\, dx = -f'(0)$

②

$n=1$ $\int_{-\infty}^{+\infty} f(x)\, \delta'(g(x))\, dx = \int_{-\infty}^{+\infty} f(x)\, \delta^{1}(g(x))\, dx$

$g(x)=y$ $\int_{-\infty}^{+\infty} f(g^{-1}(y))\, \delta'(y)\, \frac{dy}{g'(x)}$

$\pm\left(-\left(\frac{f(g^{-1}(y))}{g'(x)} \right)' \right)_{y=0,\ x=x_0}$

stop

zie REM ① PI = partial integration

x real nullpoint

combine signs

$\mp \frac{d}{dy}\left(\quad \right) = \mp \frac{d}{dy}\left(\frac{f(x)}{g'(x)} \right)_{y=0,\ x=x_0}$

rules $\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$, $\frac{d}{dy} = \frac{dx}{dy}\frac{d}{dx}$

$\frac{d}{dy}(\ ) = \frac{1}{\frac{dy}{dx}}\frac{d}{dx}(\ )$

$\frac{dy}{dx} = g'$

$\mp \frac{dx}{dy}\frac{d}{dx}\left(\frac{f(x)}{g'(x)}\right)_{x=x_0}$ $\to f$, $\to g'$

$\mp \frac{1}{\frac{dy}{dx}} \left(\frac{f'g' - f g''}{(g')^2} \right)_{x=x_0} = \mp \frac{1}{g'^2} f' - \frac{f \cdot g''}{g'^3}$

Dus

$$= \mp \left(\frac{f'(x_0)}{(g'(x_0))^2} - \frac{f(x_0)\, g''(x_0)}{(g'(x_0))^3} \right)$$

combination $=$ sign $(g'(x_0))\ (-1)^1$ zie maple code

$$\int_{-\infty}^{+\infty} f\left(g^{-1}(y)\right)\, \delta'(y)\, \frac{dy}{|g'(x)|}$$

$\delta'(y) \to \delta^{n}(y)$

↓ nth power

$$\int_{-\infty}^{+\infty} f\left(g^{-1}(y)\right)\, \delta^{n}(y)\, \frac{dy}{|g'(x)|}$$

$|g'(x)| \to g'(g^{-1}(y))$

PI n times

$$(-1)^n \left(\frac{d}{dy}\right)^n \left(\frac{f(g^{-1}(y))}{g'(g^{-1}(y))}\right)_{y=0,\ x=x_0}$$

$$\frac{d()}{dy} = \frac{dx}{dy}\frac{d()}{dx} = \frac{1}{y'}\frac{d()}{dx}$$

correctio - for slope of function $g(x)$

$\to$ " maple code

$$\operatorname{sgn}(g'(x_0))\,(-1)^n \left(\frac{1}{g'(x)}\frac{d}{dx}\right)^n \left(\frac{f(x)}{g'(x)}\right)\Bigg|_{y=0}$$

$\sum$ For all null points : $x = x_0$ $\to$ sol $\to$ " maple code, maxima code "

(add)

↓

List

see example :

Maple code, Maxima code

[-7, 7]